ESTIMATE OF LONGEVITY UNDER CONDITIONS OF HIGH-TEMPERATURE MULTICYCLE LOADING

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UDC 539.376

In different areas of machine construction, the problem of the long-term strength and longevity of structural materials under multicyclical loading is given serious attention. A very large number of papers have been published to date in which different aspects of this problem are examined (see, for example, [1-4]). In most of these papers, however, the main attention was concentrated on obtaining experimental data and the formulation of empirical dependences. Only separate papers [5-7], referring to normal temperatures, in which the longevity under multicyclical loading is estimated based on model representations, are available.

In this paper we propose a possible variant of the analytical estimate of cyclical longevity at high temperatures.

1. Basic Starting Relations. We examine a straight cylindrical rod (Fig. 1a) under an axial load P which varies with time thus:

$$P = P_m \pm P_a \Phi(ft), \tag{1.1}$$

where P is the resulting load; P_m and P_a are the amplitudes of the static and cyclical components; Φ is a function that characterizes the variation of P_a as a function of time t; f is the frequency of variation of P_a. For the multicyclical loading regime, it is characteristic that f > 1 Hz, while $\sigma_{max} = \sigma_m + |\sigma_a| < \sigma_y$, where σ_y is the yield point of the material. To solve the problem, we shall start from the fact that under multicyclical loading (1.1) at high temperatures, i.e., for $T > 0.5T_{melt}$ (T_{melt} is the melting temperature), failure of the rod can occur due to fatigue or cyclical creep, or due to their combined development [4, 8, 9]. For many structural elements, the region of mixed failure, determined by the interaction of fatigue and creep, is the predominant form. We assume below that the parameter ω, which describes the accumulation of damage, can be chosen as one of the characteristics of the state [8]. We assume that the damage will with time accumulate both as a result of creep, which is manifested in the change in the geometry of the rod and accumulation of scattered damage (Fig. 1b), and as a result of fatigue, which is manifested in the accumulation of concentrated damage in the form of a fatigue crack (Fig. 1c). As a result, the stressed state of the rod at any arbitrary moment in time t will be determined by the magnitude of the instantaneous (true) values of the stresses, which can differ greatly from the initial fixed values. Going over from the loads to the instantaneous values of the stresses and selecting for Φ a harmonic law, we rewrite the condition of loading (1.1) in the form

$$\sigma = \sigma_m + \sigma_a \sin(2\pi ft + \varphi_0), \tag{1.2}$$

where φ_0 is the initial phase angle, which is usually assumed to equal zero; σ, σ_m, σ_a are the instantaneous value of the stresses, which are determined from equations of the form

$$\sigma = \sigma_0 \psi(\varepsilon_\Sigma, \omega_y), \tag{1.3}$$

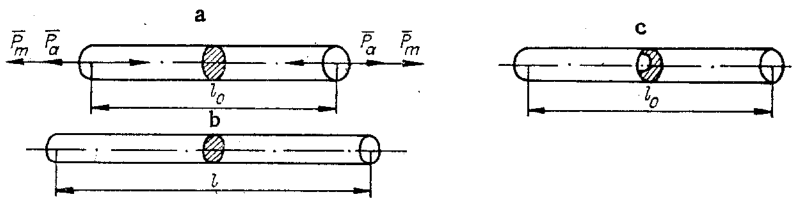

Fig. 1

Kiev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 144-148, March-April, 1984. Original article submitted January 6, 1983.

0021-8944/84/2502-0305$08.50

where σ_0, σ is the initial and instantaneous value of the stress; ε_Σ is the instantaneous value of the deformation of cyclical creep; ω_y is the instantaneous value of fatigue damage.

To solve the problem, we shall use the approach developed in [8] for determining the time of viscous-brittle fracture relative to conditions of static creep. In this case, in its simplest form the complete system of equations for uniaxial multicyclical loading with T = const can be represented in the form

$$\dot{\varepsilon}_\Sigma = \dot{\varepsilon}_\Sigma(\sigma_{m0}, \sigma_{a0}, \varepsilon_\Sigma, \omega_y); \tag{1.4}$$

$$\dot{\omega}_y = \dot{\omega}_y(\sigma_{m0}, \sigma_{a0}, \varepsilon_\Sigma, \omega_y). \tag{1.5}$$

This approach, as is well known, is called the kinetic approach, and can be extended to the three-dimensional case.

2. Longevity under Conditions of Cyclical Creep. We shall first examine the problem of determining the time up to failure only from creep, which under conditions of loading (1.2) is called cyclical, assuming that fatigue damage does not develop, i.e., $\omega_y = 0$ (Fig. 1b). In this case the rate of cyclical creep $d\varepsilon_\Sigma/dt$, according to the results in [9], is written in the form

$$d\varepsilon_\Sigma/dt = B(\sigma_m)^m(\sigma_a)^k, \tag{2.1}$$

where ε_Σ is the deformation of cyclical creep; σ_m, σ_α are the instantaneous values of the static and cyclical stresses; B, m, and k are coefficients; and, in addition, $k \simeq 0.1$-$0.5L$, which permits going from exponential dependences of ε_Σ on σ_α [9] to a power-law dependence.

Let l, F be the instantaneous and l_0, F_0 the initial length and cross-sectional area of the rod. Then $\sigma_m = P_m/F$, $\sigma_\alpha = P/F$, and $\sigma_{m0} = P_m/F_0$, $\sigma_{\alpha 0} = P_\alpha/F_0$. Assuming below that the material of the rod is incompressible, i.e., $Fl = F_0 l_0$, we find

$$\sigma_m = \sigma_{m0} l/l_0,\ \sigma_a = \sigma_{a0} l/l_0. \tag{2.2}$$

Neglecting the instantaneous deformation, the rate of cyclical creep and deformation, using the instantaneous length of the rod, can be represented in the form

$$\frac{d\varepsilon_\Sigma}{dt} = \frac{1}{l}\frac{dl}{dt},\quad \varepsilon_\Sigma = \ln\frac{l}{l_0}. \tag{2.3}$$

Substituting (2.2), (2.3) into (2.1), we obtain the differential equation of cyclical creep

$$\frac{1}{l}\frac{dl}{dt} = B(\sigma_{m0})^m(\sigma_{a0})^k\left(\frac{l}{l_0}\right)^{m+k} \tag{2.4}$$

with the initial conditions $l/l_0 = 1$ at $t = 0$. Separating variables and integrating Eq. (2.4), taking into account the initial condition, we obtain

$$t = \frac{1}{B(m+k)(\sigma_{m0})^m(\sigma_{a0})^k}\left[1 - \left(\frac{l_0}{l}\right)^{m+k}\right].$$

Assuming that the failure of the rod occurs following the model of absolute viscous failure [10], i.e., with $l = \infty$, $F = 0$, and $\sigma = \infty$, we write the time up to failure t_{pn} under conditions of cylical creep in the form

$$t_{pn} = 1/[B(m+k)(\sigma_{m0})^m(\sigma_{a0})^k]. \tag{2.5}$$

For k = 0, Eq. (2.5) coincides with the model of an absolutely viscous failure under conditions of static creep.

3. Fatigue Longevity. When the conditions for the loading (1.2) are realized, as already noted, purely fatigue failure can arise, with which the rate of development of the damage does not depend on creep, i.e., in (1.5) we assume that $\varepsilon_\Sigma = 0$. We shall consider a simplified variant of the kinetic equation of damage (1.5), assuming, in accordance with the data in [8], that

$$d\omega_y/dt = (\omega_y)^\beta v, \tag{3.1}$$

where v is the velocity of the failure front; β is a coefficient. Next, we assume that, in the presence of fatigue concentrated damage in the form of a fatigue crack (see Fig. 1c), which propagates in only one direction, i.e., $\beta = 0$, mainly forms. In this case, independent of the form of the function $\Phi(t)$ in Eq. (1.1), in determining the velocity of the failure front v we shall use the scheme in [11], with which

$$v = dl_y/dt = C_1 f(\Delta K)^n = C_1 f(\sigma_a)^n, \tag{3.2}$$

where l_y is the instantaneous length of the fatigue crack; ΔK is the peak-to-peak amplitude of the coefficient of intensity of the stresses; σ_a is the instantaneous value of the cyclical stress, which depends on l_y; C_1 and n are coefficients, of which C_1 depends on σ_m. According to the data in [12], for small values of σ_a the quantity C_1 is proportional to the static stress, i.e., in this case,

$$v = Cf(\sigma_a)^n \sigma_m,$$

where $C = C_1/\sigma_m$. We represent the instantaneous value of the amplitude of the cyclical stress σ_a at arbitrary time t in the form

$$\sigma_a = P_a/F = P_a/[F_0(1-\omega_y)] = \sigma_{a0}/(1-\omega_y) \tag{3.2'}$$

and we solve Eq. (3.2') for the parameter ω_y,

$$\omega_y = 1 - \sigma_{a0}/\sigma_a. \tag{3.3}$$

Differentiating Eq. (3.3) with respect to time and substituting the results obtained and Eq. (3.2) into the starting kinetic equation (3.1), taking into account the fact that for $\beta = 0$, $d\omega_y/dt \approx dl_y/dt$, we obtain the differential equation for fatigue damage:

$$\frac{d\sigma_a}{(\sigma_a)^{2+n}} = \frac{Cf\sigma_{m0}}{\sigma_{a0}}\,dt. \tag{3.4}$$

Integrating Eq. (3.4) taking into account the initial condition $\sigma_a = \sigma_{a0}$ at $t = 0$, we obtain

$$t = \frac{\sigma_{a0}}{Cf(1+n)\sigma_{m0}}\left[\frac{1}{(\sigma_{a0})^{1+n}} - \frac{1}{(\sigma_a)^{1+n}}\right].$$

Assuming that purely fatigue failure is brittle and that the failure of the rod occurs with $\omega_y = 1$ and $\sigma_a = \infty$, we write the time for fatigue failure t_{ff} in the form

$$t_{py} = 1/[Cf(1+n)(\sigma_{a0})^n \sigma_{m0}]. \tag{3.5}$$

The structure of Eq. (3.5) is analogous to the equation of brittle failure with creep under conditions of static loading [8].

4. Longevity Under Conditions of Interaction of Fatigue and Creep. To estimate the time of mixed failure, we shall examine the combined solution of the system of equations (1.4), (1.5). It is evident that the equation of creep will depend on the fatigue damage, while the fatigue damage will reflect the effect of creep. In this case, the true values of the stresses are determined taking into account the decrease in the area of the transverse cross section of the rod both as a result of fracturing and as a result of creep, i.e.,

$$\sigma_m = \frac{\sigma_{m0}}{1-\omega_y} e^{\varepsilon_\Sigma}, \quad \sigma_a = \frac{\sigma_{a0}}{1-\omega_y} e^{\varepsilon_\Sigma}, \tag{4.1}$$

while Eqs. (1.3), (1.4), substituting (2.1), (3.1), (3.2), and (4.1), are rewritten in the form

$$\frac{d\varepsilon_\Sigma}{dt} = \frac{B}{(1-\omega_y)^{m+k}} e^{(m+k)\varepsilon_\Sigma} (\sigma_{m0})^m (\sigma_{a0})^k; \tag{4.2}$$

$$\frac{d\omega_y}{dt} = \frac{Cf}{(1-\omega_y)^{1+n}} (\sigma_{a0})^n e^{(1+n)\varepsilon_\Sigma} \sigma_{m0}; \tag{4.3}$$

in addition, at $t = 0$, $\varepsilon_\Sigma = 0$ and $\omega_y = 0$, while at $t = t_p$, $\varepsilon_\Sigma = \infty$ and $\omega_y = 1$.

Separating variables in Eq. (4.3) and integrating with the initial condition $\omega_y = 0$ at $t = 0$, and using the expression for the time of purely fatigue failure (3.5), we obtain

$$1-\omega_y=\left[1-\frac{e^{(1+n)\varepsilon_\Sigma}(2+n)\,t}{(1+n)\,t_{py}}\right]^{\frac{1}{2+n}}. \quad (4.4)$$

The differential equation of cyclical creep (4.2), taking into account (4.4), is written in the form

$$\frac{d\varepsilon_\Sigma}{dt}=\frac{Be^{(m+k)\varepsilon_\Sigma}}{\left[1-\frac{(2+n)\,e^{(1+n)\varepsilon_\Sigma}t}{(1+n)\,t_{py}}\right]^{\frac{m+k}{2+n}}}(\sigma_{m0})^m(\sigma_{a0})^k. \quad (4.5)$$

To obtain the solution in an explicit form, we shall restrict our attention to conditions under which creep does not affect the formation of cracks, i.e., we assume in (4.5) that $(1 + n)\varepsilon_\Sigma = 0$. The variables in (4.5) separate in this case, and integrating with the initial condition $\varepsilon_\Sigma = 0$ at $t = 0$, using the expression for the time of failure as a function of the cyclical creep (2.5), we obtain

$$\frac{t}{t_{py}}=\frac{1+n}{2+n}\left\{1-\left[1+\frac{(m+k-n-2)\left(1-e^{-(m+k)\varepsilon_\Sigma}\right)t_{pn}}{(1+n)(2+n)\,t_{py}}\right]^{-\frac{n+2}{m+k-n-2}}\right\}. \quad (4.6)$$

To determine the time of mixed failure, $t_{p\Sigma}$, we set in Eq. (4.6) $\varepsilon_\Sigma = \infty$, from where

$$\frac{t_{p\Sigma}}{t_{py}}=\frac{1+n}{2+n}\left\{1-\left[1+\frac{(m+k-n-2)\,t_{pn}}{(1+n)(2+n)\,t_{py}}\right]^{-\frac{2+n}{m+k-n-2}}\right\}. \quad (4.7)$$

It is evident from the structure of Eq. (4.7) that creep decreases the longevity of the rod, found from the scheme of purely fatigue failure, since $t_{p\Sigma} < t_{py}$. Formula (4.7) is valid for $\sigma \neq 0$ and in the opposite case purely viscous failure occurs. For $\sigma_m = 0$, the time of purely fatigue failure is determined from Eq. (4.7).

We shall examine the relation between the longevity with mixed failure and the longevity under conditions of creep development. For this we separate variables in (4.2) and integrate with the initial condition $\varepsilon_\Sigma = 0$ at $t = 0$. Taking into account the expression for the time of failure as a result of cyclical creep (2.5), we obtain

$$e^{(m+k)\varepsilon_\Sigma}=\left[1-\frac{t}{(1-\omega_y)^{m+k}\,t_{pn}}\right]^{-1}. \quad (4.8)$$

The differential equation for the damage parameter (4.3), taking into account (4.8), is written in the form

$$\frac{d\omega_y}{dt}=\frac{Cf}{(1-\omega_y)^{1+n}}(\sigma_{a0})^n\sigma_{m0}\left[1-\frac{1}{(1-\omega_y)^{m+k}\,t_{pn}}\right]^{-\frac{1+n}{m+k}}. \quad (4.9)$$

To obtain the solution in an explicit form, we again restrict our attention to conditions under which creep does not affect the accumulation of damage, i.e., we assume in (4.9) that $m + k = 0$. The variables in (4.9) separate in this case and, integrating with the initial condition $\omega_y = 0$ at $t = 0$, using the expression for the time of fatigue failure (3.5), we obtain

$$\frac{t_p}{t_{pn}}=1-\left\{1+\frac{(1+n)(1+n-m-k)}{(2+n)(m+k)}\left[1-(1-\omega_y)^{2+n}\right]\frac{t_{py}}{t_{pn}}\right\}^{-\frac{m+k}{1+n-m-k}}. \quad (4.10)$$

To determine the time of mixed failure $t_{p\Sigma}$, we set in Eq. (4.10) $\omega_y = 1$ at $t = t_{p\Sigma}$, i.e.,

$$\frac{t_{p\Sigma}}{t_{pn}} = 1 - \left[1 + \frac{(1+n)(1+n-m-k)\,t_{py}}{(2+n)(m+k)\,t_{pn}}\right]^{-\frac{m+k}{1+n-m-k}}. \tag{4.11}$$

It is evident from the structure of Eq. (4.11) that fatigue damage decreases the longevity of the rod, found from the condition of failure due to creep, since $t_{p\Sigma} < t_{pn}$.

As an example, we shall calculate the time up to failure of samples of heat-resistant nickel alloy at 800°C, tested under the loading conditions (1.1). Figure 2 shows the experimental points and the approximating line (continuous curve) according to the data in [4]. The dashed curve was calculated using Eq. (4.7). Here we used the following starting data: $\sigma_m = 20$ kg/mm², $f = 50$ Hz, $\log B = -12$, $C = 1.52 \cdot 10^{-8}$, $k = 6.3$, $m = 7.5$, $n = 2.2$. It is evident that on the whole the agreement between the experimental results and the calculation is satisfactory (the maximum error does not exceed 15%).

We shall also examine the possibility of using Eqs. (4.7) and (4.11) to calculate the curves of equal longevity, whose well-known form is given in terms of the diagram of limiting stresses in the coordinates $\sigma_a - \sigma_m$ [3, 4, 8]. In this case, taking into account the fact that for curves of equal longevity $t_{py} = t_{pn}$, and solving Eqs. (4.7) and (4.11) for σ_a, we obtain the following equations for the diagrams of limiting stresses:

$$\sigma_{a0} = B_1 (\sigma_{m0})^{-\frac{1}{n}} (t_{p\Sigma})^{-\frac{1}{n}} [Cf(2+n)]^{-\frac{1}{n}}; \tag{4.12}$$

$$\sigma_{a0} = B_2 (\sigma_{m0})^{-\frac{m}{k}} (t_{p\Sigma})^{-\frac{1}{k}} [B(m+k)]^{-\frac{1}{k}}, \tag{4.13}$$

where

$$B_1 = \left\{1 - \left[1 + \frac{m+k-n-2}{(1+n)(2+n)}\right]^{-\frac{2+n}{m+k-n-2}}\right\}^{\frac{1}{n}};$$

$$B_2 = \left\{1 - \left[1 + \frac{(1+n)(1+n-m-k)}{(2+n)(m+k)}\right]^{-\frac{m+k}{1+n-m-k}}\right\}^{\frac{1}{k}}.$$

It is evident from the structure of Eqs. (4.12) and (4.13) that as the time to failure increases or as one of the components of the limiting stress increases, the other component decreases. In contrast to the well-known [1, 3, 4, 12] equations, Eqs. (4.12) and (4.13), which contain as one of the parameters the time up to failure, permit calculating the limiting stresses, without being tied to experimental data on fatigue resistance with a symmetrical cycle and on the resistance of prolonged static strength.

As an example, we shall calculate the diagrams of limiting stresses for the alloy ÉI867 at 900°C, the experimental data for which are taken from [12]. Figure 3 shows the experimental data (points) and diagrams, calculated using Eq. (4.13) (dashed lines) for longevities of 1, 10, and 100 h (points 1-3, respectively). In the calculation, we used the following

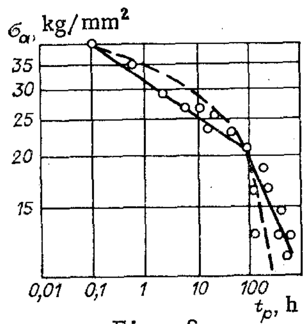

Fig. 2

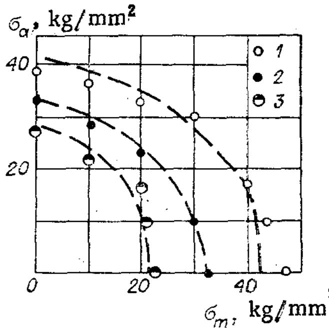

Fig. 3

values of the coefficients: $C = 2.2 \cdot 10^{-11}$, $\log B = -11.2$, $m = 7.2$, $k = 0.5$, $n = 4.2$. A comparison shows that the computed diagrams agree satisfactorily with the experimental data (the error does not exceed 12%).

Thus the general approach of Yu. N. Rabotnov to estimating the time of viscous-brittle fracture under conditions of static creep permits solving the problem of calculating the longevity of heat-resistant materials under multicyclical loading. It was shown that the longevity under conditions of interaction of fatigue and creep is shorter than under conditions of only fatigue or only creep.

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